Hamiltonian Decomposition of Recursive Circulants*

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Abstract. We show that recursive circulant $G(cd^m, d)$ is hamiltonian decomposable. Recursive circulant is a graph proposed for an interconnection structure of multicomputer networks in [8]. The result is not only a partial answer to the problem posed by Alspach that every connected Cayley graph over an abelian group is hamiltonian decomposable, but also an extension of Micheneau's that recursive circulant $G(2^m, 4)$ is hamiltonian decomposable.

1 Introduction

We say a graph G is *hamiltonian decomposable* if either the degree of G is $2k$ and the edges of G can be partitioned into k hamiltonian cycles, or the degree of G is $2k+1$ and the edges of G can be partitioned into k hamiltonian cycles and a 1-factor, where a 1-factor of a graph is a 1-regular spanning subgraph. If G is hamiltonian decomposable then G is loopless, connected, and regular.

It is necessary for a graph G to have a hamiltonian decomposition that G has a hamiltonian cycle. However, the condition is by far not sufficient. Many authors have been dealing with sufficient conditions for the existence of a decomposition of a graph into hamiltonian cycles. But still, the current status of the matter lies, for the most part, in the sphere of problems and conjectures.

A survey on hamiltonian decomposition of graphs is provided in [3, 4]. The complete graph K_n with odd (resp. even) number n of vertices is decomposable into hamiltonian cycles (resp. paths). The complete k-partite graph $K(n_1, n_2, \cdots, n_k)$ is hamiltonian decomposable if and only if $n_1 = n_2 = \cdots = n_k$. The following problem on hamiltonian decomposability of Cayley graphs is posed by Alspach [1].

Problem 1 *Does every connected Cayley graph over an abelian group have a hamiltonian decomposition?*

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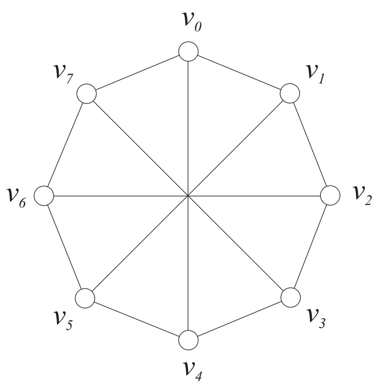
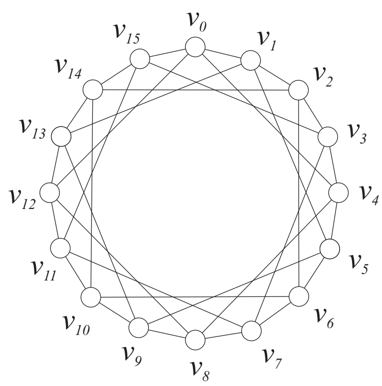
Much of the focus of research has been directed towards proving special cases of the problem. The answer is yes when the degree of the graph is five or less. The product of any number of cycles $C_{n_1} \times C_{n_2} \times \cdots \times C_{n_k}$ is hamiltonian decomposable. The product of cycles is isomorphic to the Cayley graph of the corresponding cyclic groups with the standard generating set. The m-cube Q_m, the Cayley graph of the product of m copies of a cyclic group Z_2, is hamiltonian decomposable.

Recursive circulant is a graph proposed for an interconnection structure of multicomputer networks in [8]. The recursive circulant $G(N,d)$, $d \geq 2$, is defined as follows: the vertex set $V = \{v_0, v_1, v_2, \cdots, v_{N-1}\}$, and the edge set $E = \{(v_i, v_j) \mid \text{there exists } k,\ 0 \leq k \leq \lceil \log_d N \rceil - 1, \text{ such that } i + d^k \equiv j \pmod{N}\}$. Here each d^k is called a *jump*, and the *size* of the edge (v_i, v_j) is d^k. $G(N,d)$ also can be defined as a circulant graph with N vertices and jumps of powers of d, $d^0, d^1, \cdots, d^{\lceil \log_d N \rceil - 1}$. Examples of $G(N,d)$ are shown in Fig. 1.



(a) $G(8,4)$



(b) $G(16,4)$

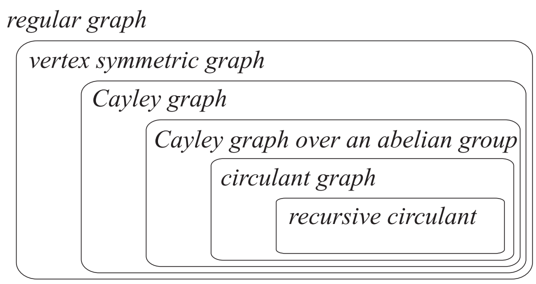
Fig. 1. Examples of $G(N,d)$

Recursive circulant is a Cayley graph over an abelian group, in more precise words, the Cayley graph of the cyclic group Z_N with the generating set $\{d^0, d^1, \cdots, d^{\lceil \log_d N \rceil - 1}\}$. This paper is concerned with hamiltonian decomposability of recursive circulants. Micheneau shows that recursive circulant $G(N,d)$ with $N = 2^m$ and $d = 4$ is hamiltonian decomposable [7]. We shall prove that $G(N,d)$ has a hamiltonian decomposition when $N = cd^m$, $1 \leq c < d$ for some integer c and d. The result is an extension of Micheneau's as well as a progress on Alspach's problem.

From now on, all arithmetics are done modulo cd^m using the appropriate residues. Graph theoretic terms not defined here can be found in [2]. This paper is organized as follows. We give some preliminaries and related works in Section 2, and prove the main theorem in Section 3 that $G(cd^m, d)$ is hamiltonian decomposable. Finally, concluding remarks are given in Section 4.

2 Preliminaries and Related Works

Let us consider the classes of graphs containing recursive circulants. Recursive circulant $G(N, d)$ is a circulant graph. A circulant graph is defined as a Cayley graph over a cyclic group. Every Cayley graph over a general group is vertex symmetric, and thus regular. These inclusion relationships are shown in Fig. 2 (a). Depending on restriction to N and d, the recursive circulants also have inclusion relationships shown in Fig. 2 (b).



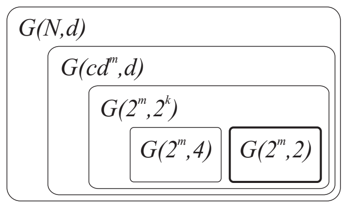


Fig. 2. Graph classes

Hamiltonian decomposition is one of the most interesting strong hamiltonicity, that is, a hamiltonian property which implies the existence of a hamiltonian cycle. Hamiltonian connectedness is also a strong hamiltonian property. A graph is *hamiltonian connected* if there is a hamiltonian path joining every pair of vertices in the graph.

When one is trying to prove that every graph in a class has a certain hamiltonian property and fails to do so, then typically two approaches are taken. One is to restrict the class of graphs and prove that the property holds over the restricted class, while the other is to restrict the property and prove that every graph in the class satisfies the restricted property. Many researches on hamiltonicity of graphs in the literature take these approaches. Some well-known conjectures and impressive properties on hamiltonicity of graphs in the classes that we have interest are shown below.

Conjecture 2 *(a) Every connected vertex symmetric graph has a hamiltonian path[Lovasz].*

(b) Every connected Cayley graph with three or more vertices has a hamiltonian cycle[Chen].
(c) Every $2k$-regular connected Cayley graph on a finite abelian group is hamiltonian decomposable[Alspach].

Theorem 3 *(a) Every connected Cayley graph over an abelian group is hamiltonian connected [5].*
(b) Recursive circulant $G(2^m, 4)$ is hamiltonian decomposable [7].

Recursive circulant $G(N, d)$ with three or more vertices has a hamiltonian cycle. Obviously, the set of edges of size 1 in $G(N, d)$ form a hamiltonian cycle. Theorem 3 (a) shows that $G(N, d)$ has a strong hamiltonicity of hamiltonian connectedness.

Recursive circulant $G(N, d)$ has a recursive structure when $N = cd^m$, $1 \leq c < d$. In other words, $G(cd^m, d)$ can be defined recursively by utilizing the following property.

Property 4 *Let V_i be a subset of vertices in $G(cd^m, d)$ such that $V_i = \{v_j \mid j \equiv i \pmod{d}\}$, $m \geq 1$. For $0 \leq i \leq d-1$, the subgraph of $G(cd^m, d)$ induced by V_i is isomorphic to $G(cd^{m-1}, d)$.*

$G(cd^m, d)$, $m \geq 1$, can be constructed recursively on d copies of $G(cd^{m-1}, d)$ as follows. Let $G_i(V_i, E_i)$, $0 \leq i \leq d-1$, be a copy of $G(cd^{m-1}, d)$. We assume that $V_i = \{v_0^i, v_1^i, \cdots, v_{cd^{m-1}-1}^i\}$, and G_i is isomorphic to $G(cd^{m-1}, d)$ with the isomorphism mapping v_j^i to v_j. We relabel v_j^i by v_{jd+i}. The vertex set V of $G(cd^m, d)$ is $\bigcup_{0 \leq i \leq d-1} V_i$, and the edge set E is $\bigcup_{0 \leq i \leq d-1} E_i \cup X$, where $X = \{(v_j, v_{j'}) \mid j + 1 \equiv j' \pmod{cd^m}\}$. The construction of $G(32, 4)$ on four copies of $G(8, 4)$ is illustrated in Fig. 3. Every edge in X is of size 1, and X forms a hamiltonian cycle, called the *fundamental hamiltonian cycle*, in $G(cd^m, d)$.

We denote by δ_m the degree of $G(cd^m, d)$. The degree of a graph is the minimum degree over all vertices in the graph. δ_m is greater than δ_{m-1} by two, that is, $\delta_m = \delta_{m-1} + 2$, $m \geq 1$. δ_m in a closed-form is shown below.

$$\delta_m = \begin{cases} 2m-1 & \text{if } c = 1 \text{ and } d = 2; \\ 2m & \text{if } c = 1 \text{ and } d \neq 2; \\ 2m+1 & \text{if } c = 2; \\ 2m+2 & \text{if } c > 2. \end{cases}$$

3 Hamiltonian Decomposition of $G(cd^m, d)$

In this section, we prove the following main theorem by mathematical induction on degree of recursive circulant $G(cd^m, d)$. We have two cases depending on the size of d.

Theorem 5 *Recursive circulant $G(cd^m, d)$ is hamiltonian decomposable.*

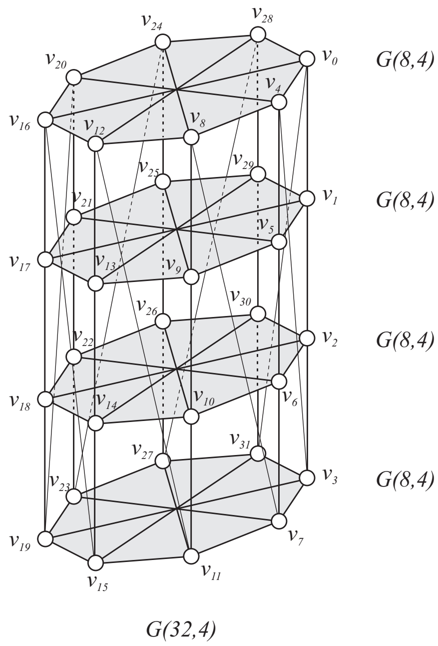


Fig. 3. Recursive structure of $G(32, 4)$

3.1 Case of $d \geq 4$

We show that $G(cd^m, d)$ has a hamiltonian decomposition such that hamiltonian cycles in the decomposition satisfy the following two conditions C1.1 and C1.2.

C1.1 For every hamiltonian cycle C_j in the decomposition, there exists an index k_j such that C_j passes through two adjacent edges (v_{k_j}, v_{k_j+1}) and (v_{k_j+1}, v_{k_j+2}) of size 1.

C1.2 For any pair C_j and $C_{j'}$ of hamiltonian cycles in the decomposition, $\{v_{k_j}, v_{k_j+1}, v_{k_j+2}\} \cap \{v_{k_{j'}}, v_{k_{j'}+1}, v_{k_{j'}+2}\} = \emptyset$.

Let us first consider a hamiltonian decomposition of $G(cd^m, d)$ with small degree. $G(cd^m, d)$ with degree two is the fundamental hamiltonian cycle itself. When the degree of $G(cd^m, d)$ is three, $G(cd^m, d)$ consists of the fundamental hamiltonian cycle and a 1-factor. In both cases, the natural decomposition leads to a hamiltonian decomposition satisfying the above two conditions.

Now we try to find a hamiltonian decomposition of $G(cd^m, d)$ by using the fact that $G(cd^{m-1}, d)$ has a hamiltonian decomposition satisfying the two conditions. The degree of $G(cd^m, d)$ is greater than that of $G(cd^{m-1}, d)$ by two. Note that $G(cd^m, d)$ has a recursive structure, that is, $G(cd^m, d)$ is constructed on d copies of $G(cd^{m-1}, d)$. We let G_i denote a copy of $G(cd^{m-1}, d)$, and assume that the vertices of G_i is $\{v_0^i, v_1^i, \cdots, v_{cd^{m-1}-1}^i\}$. The vertex v_j^i is relabeled by v_{jd+i}. Hereafter, two labels for the vertex are used interchangeably.

We denote by X the fundamental hamiltonian cycle in $G(cd^m, d)$. Let C_j^i denote a hamiltonian cycle in the hamiltonian decomposition of G_i. C_j^i passes through the edges $(v_{k_j}^i, v_{k_j+1}^i)$ and $(v_{k_j+1}^i, v_{k_j+2}^i)$ for some k_j by condition C1.1.

The basic idea of the proof is that we merge d different cycles C_j^i, $0 \le i < d$, into one hamiltonian cycle C_j in $G(cd^m, d)$ by exchanging some edges of C_j^i's with edges in X while we keep X being a hamiltonian cycle. We merge another d different cycles $C_{j'}^i$ into another hamiltonian cycle $C_{j'}$ by these edge exchange operations. We repeat this until a hamiltonian decomposition of $G(cd^m, d)$ is obtained.

We represent C_j^i and X as a sequence of vertices. It is convenient to represent explicitly only the vertices concerned with edge exchange operations as follows. Here P^i is a path from $v_{k_j+2}^i$ to $v_{k_j}^i$ (excluding the start and end vertices) passing through all the vertices in G_i except $v_{k_j}^i$, $v_{k_j+1}^i$, and $v_{k_j+2}^i$. P is a path from $v_{k_j+2}^{d-1}$ to $v_{k_j}^0$ in $G(cd^m, d)$ passing through all the other vertices not represented explicitly. In other words, P passes through the vertices $\{v_j^i \,|\, 0 \le i < d, 0 \le j < cd^{m-1}, j \ne k_j, k_j+1, k_j+2\}$ in some order.

$$C_j^i = v_{k_j}^i, v_{k_j+1}^i, v_{k_j+2}^i, P^i$$
$$X = v_{k_j}^0, v_{k_j}^1, \cdots, v_{k_j}^{d-1}, v_{k_j+1}^0, v_{k_j+1}^1, \cdots, v_{k_j+1}^{d-1}, v_{k_j+2}^0, v_{k_j+2}^1, \cdots, v_{k_j+2}^{d-1}, P$$

The edge exchange operation depends on the parity of d.

Case 1 even d

d hamiltonian cycles C_j^i's are merged into a hamiltonian cycle C_j in $G(cd^m, d)$ as follows(See Fig. 4 (a)). Note that X still remains a hamiltonian cycle in $G(cd^m, d)$.

$$\begin{aligned} C_j = {} & v_{k_j}^0, v_{k_j}^1, P^1, v_{k_j+2}^1, \cdots, v_{k_j+2}^{d-4}, P^{d-4}, v_{k_j}^{d-4}, v_{k_j}^{d-3}, P^{d-3}, v_{k_j+2}^{d-3}, v_{k_j+2}^{d-2}, \\ & P^{d-2}, v_{k_j}^{d-2}, v_{k_j+1}^{d-2}, v_{k_j+1}^{d-3}, v_{k_j+1}^{d-4}, \cdots, v_{k_j+1}^1, v_{k_j+1}^0, v_{k_j}^{d-1}, P^{d-1}, \\ & v_{k_j+2}^{d-1}, v_{k_j+1}^{d-1}, v_{k_j+2}^0, P^0 \\ X = {} & v_{k_j}^0, v_{k_j+1}^0, v_{k_j+2}^0, v_{k_j+2}^1, v_{k_j+1}^1, v_{k_j}^1, \cdots, v_{k_j}^{d-4}, v_{k_j+1}^{d-4}, v_{k_j+2}^{d-4}, v_{k_j+2}^{d-3}, \\ & v_{k_j+1}^{d-3}, v_{k_j}^{d-3}, v_{k_j}^{d-2}, v_{k_j}^{d-1}, v_{k_j+1}^{d-1}, v_{k_j+1}^{d-2}, v_{k_j+2}^{d-2}, v_{k_j+2}^{d-1}, P \end{aligned}$$

The edge exchange operations can be performed independently since the hamiltonian decomposition of G_i always satisfies the condition C1.2. Thus every hamiltonian cycle in G_i can be merged into a hamiltonian cycle in $G(cd^m, d)$. If G_i has a 1-factor in the hamiltonian decomposition, G_i has odd degree and even number of vertices. The union of 1-factors in G_i's forms a 1-factor of $G(cd^m, d)$. We have successfully constructed the hamiltonian decomposition of $G(cd^m, d)$ in this case.

Now let us prove that the hamiltonian decomposition presented satisfies the conditions C1.1 and C1.2. C_j passes through the edges $(v_{k_j+1}^0, v_{k_j+1}^1)$ and $(v_{k_j+1}^1, v_{k_j+1}^2)$. Eventually in $G(cd^m, d)$, they are adjacent edges of size 1. X

passes through the edges $(v_{k_j}^{d-3}, v_{k_j}^{d-2})$ and $(v_{k_j}^{d-2}, v_{k_j}^{d-1})$. X also satisfies the condition C1.1. The three vertices associated with C_j are different from the three vertices associated with X. From the fact that any pair C_j^i and $C_{j'}^i$ of hamiltonian cycles in the decomposition of G^i satisfies the condition C1.2, the three vertices $\{v_{k_j+1}^0, v_{k_j+1}^1, v_{k_j+1}^2\}$ associated with C_j are disjoint from the three vertices $\{v_{k_{j'}+1}^0, v_{k_{j'}+1}^1, v_{k_{j'}+1}^2\}$ associated with $C_{j'}$. Thus we have a hamiltonian decomposition of $G(cd^m, d)$ satisfying C1.1 and C1.2.

Case 2 odd d

The proof of this case is similar to that of Case 1. C_j and X after the edge exchange operation are shown below(See Fig. 4 (b)).

$$C_j = v_{k_j}^0, v_{k_j}^1, P^1, v_{k_j+2}^1 v_{k_j+2}^2, P^2, v_{k_j}^2, \cdots, v_{k_j}^{d-2}, P^{d-2}, v_{k_j+2}^{d-2}, v_{k_j+2}^{d-1}, \\ P^{d-1}, v_{k_j}^{d-1}, v_{k_j+1}^{d-1}, v_{k_j+1}^{d-2}, \cdots, v_{k_j+1}^2, v_{k_j+1}^1, v_{k_j+1}^0, v_{k_j+2}^0, P^0$$

$$X = v_{k_j}^0, v_{k_j+1}^0, v_{k_j}^{d-1}, v_{k_j}^{d-2}, v_{k_j+1}^{d-2}, v_{k_j+2}^{d-2}, \cdots, v_{k_j+2}^2, v_{k_j+1}^2, v_{k_j}^2, v_{k_j}^1, \\ v_{k_j+1}^1, v_{k_j+2}^1, v_{k_j+2}^0, v_{k_j+1}^{d-1}, v_{k_j+2}^{d-1}, P$$

C_j and X are hamiltonian cycles in $G(cd^m, d)$. C_j passes through edges $(v_{k_j+1}^0, v_{k_j+1}^1)$ and $(v_{k_j+1}^1, v_{k_j+1}^2)$, and X passes through $(v_{k_j+1}^{d-1}, v_{k_j+2}^0)$ and $(v_{k_j+2}^0, v_{k_j+2}^1)$. It is easy to check that the hamiltonian decomposition presented satisfies the conditions C1.1 and C1.2, and thus omitted here.

3.2 Case of $d = 2, 3$

We show that $G(cd^m, d)$ has a hamiltonian decomposition which satisfies the following two conditions C2.1 and C2.2.

C2.1 For every hamiltonian cycle C_j in the decomposition, there exists an index k_j such that C_j passes through an edge (v_{k_j}, v_{k_j+1}) of size 1.

C2.2 For any pair C_j and $C_{j'}$ of hamiltonian cycles in the decomposition, $\{v_{k_j}, v_{k_j+1}\} \cap \{v_{k_{j'}}, v_{k_{j'}+1}\} = \emptyset$.

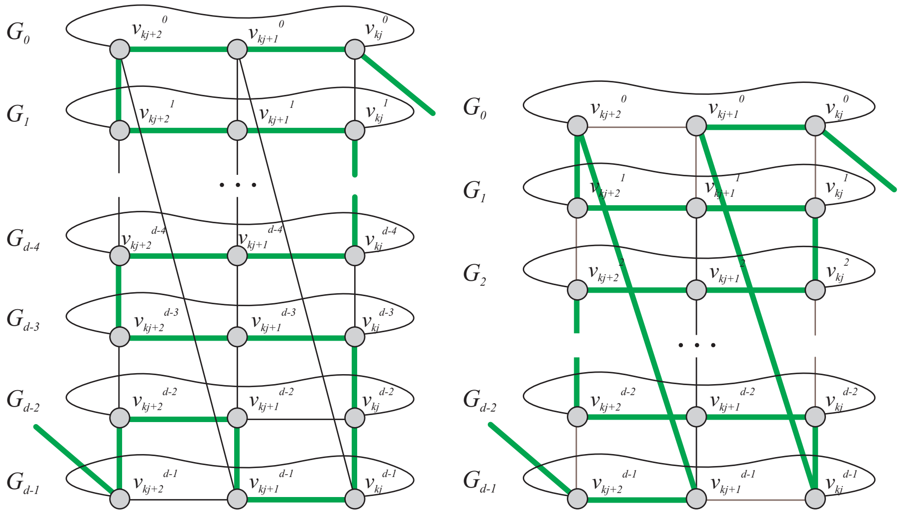
We observe that $G(cd^m, d)$ with degree two or three has a hamiltonian decomposition satisfying conditions C1.1 and C1.2, and that the decomposition also satisfies the above two conditions C2.1 and C2.2. We have two cases.

Case 1 $d = 3$

C_j^i and X are shown in the following. Here P^i is a path from $v_{k_j+1}^i$ to $v_{k_j}^i$ passing through all the vertices in G^i excluding the start and end vertices. P is a path from $v_{k_j+1}^2$ to $v_{k_j}^0$ passing through all the vertices $\{v_j^i \mid 0 \le i < d, 0 \le j < cd^{m-1}, j \ne k_j, k_j + 1\}$ in $G(cd^m, d)$.

$$C_j^i = v_{k_j}^i, v_{k_j+1}^i, P^i$$

$$X = v_{k_j}^0, v_{k_j}^1, v_{k_j}^2, v_{k_j+1}^0, v_{k_j+1}^1, v_{k_j+1}^2, P$$



(a) even d (b) odd d

Fig. 4. Case of $d \geq 4$

C_j and X after the edge exchange operation are as follows(See Fig. 5 (a)).

$$C_j = v^0_{k_j}, v^1_{k_j}, P^1, v^1_{k_j+1}, v^2_{k_j+1}, P^2, v^2_{k_j}, v^0_{k_j+1}, P^0$$
$$X = v^0_{k_j}, v^0_{k_j+1}, v^1_{k_j+1}, v^1_{k_j}, v^2_{k_j}, v^2_{k_j+1}, P$$

C_j passes through edge $(v^0_{k_j}, v^1_{k_j})$ of size 1, and X passes through edge $(v^0_{k_j+1}, v^1_{k_j+1})$. The vertices $\{v^0_{k_j}, v^1_{k_j}\}$ associated with C_j are disjoint from the vertices $\{v^0_{k_{j'}+1}, v^1_{k_{j'}+1}\}$ associated with any other hamiltonian cycle $C_{j'}$. Thus the hamiltonian decomposition satisfies the conditions C2.1 and C2.2.

Case 2 $d = 2$

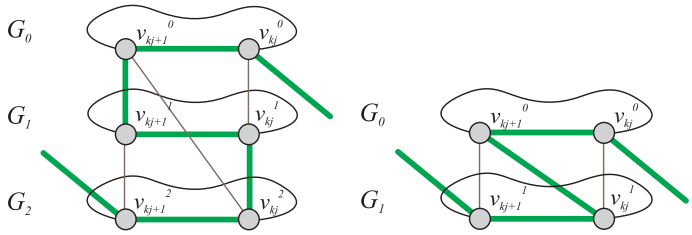
C^i_j and X are shown in the following. P^i and P can be defined in a similar way to Case 1.

$$C^i_j = v^i_{k_j}, v^i_{k_j+1}, P^i$$
$$X = v^0_{k_j}, v^1_{k_j}, v^0_{k_j+1}, v^1_{k_j+1}, P$$

C_j and X after the edge exchange operation are as follows(See Fig. 5 (b)).

$$C_j = v^0_{k_j}, v^1_{k_j}, P^1, v^1_{k_j+1}, v^0_{k_j+1}, P^0$$
$$X = v^0_{k_j}, v^0_{k_j+1}, v^1_{k_j}, v^1_{k_j+1}, P$$

C_j passes through edge $(v^0_{k_j}, v^1_{k_j})$ of size 1. But all edges in X associated with the edge exchange operation are not of size 1. If we can choose a vertex v^0_h in G_0 such that $v^0_h \neq v^0_{k_j}, v^0_{k_j+1}$ over all cycles C^0_j in the hamiltonian decomposition of G_0, then the edge (v^0_h, v^1_h) must be in X and of size 1. The existence of such a vertex can be shown by a counting argument. G_0 is a copy of $G(cd^{m-1}, d)$ with $d = 2$, that is, $G(2^{m-1}, 2)$. $G(2^{m-1}, 2)$ has degree $2(m-1) - 1 = 2m - 3$ and $\lfloor (2m-3)/2 \rfloor = m - 2$ hamiltonian cycles in the decomposition. Two vertices $v^0_{k_j}$ and $v^0_{k_j+1}$ are associated with each hamiltonian cycle C^0_j and $2(m-2)$ vertices in total. The number $2(m-2)$ is smaller than the number 2^{m-1} of vertices in $G(2^{m-1}, 2)$. Thus, we can always choose such a vertex v^0_h in G_0. Obviously, the hamiltonian decomposition satisfies the condition C2.2.



(a) $d = 3$ (b) $d = 2$

Fig. 5. Case of $d = 2, 3$

4 Concluding Remarks

We have shown that recursive circulant $G(cd^m, d)$ is hamiltonian decomposable. A hamiltonian decomposition can be constructed recursively by following the proof given in this paper. Associated with the Alspach's problem, we give conjectures on hamiltonian decomposability of recursive circulants and circulant graphs.

Conjecture 6 *(a) Recursive circulant $G(N, d)$ is hamiltonian decomposable.* *(b) Every connected circulant graph is hamiltonian decomposable.*

A directed version of the hamiltonian decomposition problem, called *dihamiltonian decomposition*, is decomposing a graph G into directed hamiltonian cycles, when we regard an edge (v, w) of G as two directed edges $\langle v, w \rangle$ and $\langle w, v \rangle$ of opposite direction. The problem has an application in the design of reliable algorithms for communication problems such as broadcasting and multicasting under the wormhole routing model [6]. We pose open problems on dihamiltonian decomposability of recursive circulants and m-cubes. Both recursive circulant $G(8,4)$ and 3-cube Q_3 are not dihamiltonian decomposable due to the fact that every 3-regular graph with a multiple of 4 vertices is not dihamiltonian decomposable [9].

Problem 7 *(a) Does recursive circulant $G(2^m, 4)$ with $m \geq 4$ have a dihamiltonian decomposition?*
(b) Does m-cube Q_m with $m \geq 4$ have a dihamiltonian decomposition?

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